

Optimization of Energy Consumption Control of Water Temperature in the Tank of the Smart Home System

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Annotation. The article examines the optimization of energy consumption in the water temperature control system in a smart home tank using modern mathematical methods controlled in the Blynk application. The main attention is paid to the integration of cubic splines and PID controllers into the mathematical model of the system. The problem statement and the manual solution of the differential equation describing the dynamics of water temperature are given. The results obtained confirm that this method allows not only to accurately control the temperature regime, but also to significantly reduce the energy consumption of the system.

Keywords. Optimization of energy consumption, temperature control, smart home system, cubic splines, PID controller, mathematical modeling.

Introduction. To ensure a comfortable environment, users need innovative smart home systems that can include optimizing energy consumption. One of the most urgent tasks is to control the temperature of the water in the tank, which requires reliable and precise regulation. The article discusses in detail the process of integrating cubic splines into the solution of a differential equation describing thermal processes in a container. Special attention is paid to the boundary conditions of smoothness and coincidence of the function and its derivatives at the nodal points. As a result, a comprehensive solution to the problem of optimizing energy consumption is proposed, based on precise mathematical methods and modern control algorithms [3,5,6].

Setting the task. The task is to minimize energy consumption to maintain a set water temperature in the tank of the smart home system. To do this, it is necessary to develop a mathematical model that takes into account the heat exchange between water and the environment, as well as the dynamics of heating and cooling water [7]. Further, using PID controllers and splines, it is possible to optimize the control of water temperature [2,8].

A mathematical model.

Consider a tank with water volume V , mass m and specific heat capacity c_p . The temperature of the water in the tank is denoted by $T_w(t)$, and the ambient temperature is $T_a(t)$. The input power of the heater is $P(t)$.

The basic equation of heat transfer [1,4]:

$$mc_p \frac{dT_w(t)}{dt} = P(t) - Q(t) \quad (1)$$

here $Q(t)$ is the heat lost by water due to heat exchange with the environment.

Newton's law of cooling can be used for heat transfer:

$$Q(t) = k(T_w(t) - T_a(t)) \quad (2)$$

where k is the heat transfer coefficient.

Thus, the differential equation takes the form:

$$mc_p \frac{dT_w(t)}{dt} = P(t) - k(T_w(t) - T_a(t)) \quad (3)$$

To approximate the function of ambient temperature $T_a(t)$ and heater power $P(t)$, we use cubic splines. Let's say we have a set of points $(t_i, T_a(t_i))$ and $(t_i, P(t_i))$. The cubic spline $S(t)$ is defined at each interval $[t_i, t_{i+1}]$:

$$S_i(t) = a_i + b_i(t - t_i) + c_i(t - t_i)^2 + d_i(t - t_i)^3 \quad (4)$$

where the coefficients a_i, b_i, c_i, d_i are determined from the conditions of smoothness and coincidence of the function and its derivatives at the nodal points.

Decision.

Consider equation (3):

$$mc_p \frac{dT_w(t)}{dt} = P(t) - k(T_w(t) - T_a(t))$$

Let $T_a(t)$ and $P(t)$ be approximated by cubic splines:

$$T_a(t) = S_a(t) = a_i + b_i(t - t_i) + c_i(t - t_i)^2 + d_i(t - t_i)^3, \quad (5)$$

$$P(t) = S_p(t) = A_i + B_i(t - t_i) + C_i(t - t_i)^2 + D_i(t - t_i)^3 \quad (6)$$

Let's substitute these expressions into the equation:

$$mc_p \frac{dT_w(t)}{dt} = S_p(t) - k(T_w(t) - S_a(t)) \quad (7)$$

Let's bring the equation to the standard form:

$$mc_p \frac{dT_w(t)}{dt} + kT_w(t) = S_p(t) + kS_a(t) \quad (8)$$

This is a linear differential equation of the first order [2,10]. Let's solve it using the integrating multiplier method.

Integrating multiplier:

$$\mu(t) = e^{\int \frac{k}{mc_p} dt} = e^{\frac{k}{mc_p} t} \quad (9)$$

Multiply the equation by $\mu(t)$:

$$e^{\frac{k}{mc_p} t} \frac{dT_w(t)}{dt} + \frac{k}{mc_p} e^{\frac{k}{mc_p} t} T_w(t) = e^{\frac{k}{mc_p} t} S_p(t) + k e^{\frac{k}{mc_p} t} S_a(t) \quad (10)$$

The left side of the equation is represented as the full derivative:

$$\frac{d}{dt} \left(e^{\frac{k}{mc_p} t} T_w(t) \right) = e^{\frac{k}{mc_p} t} S_p(t) + k e^{\frac{k}{mc_p} t} S_a(t) \quad (11)$$

Let's integrate both parts of the equation:

$$e^{\frac{k}{mc_p} t} T_w(t) = \int \left(e^{\frac{k}{mc_p} t} S_p(t) + k e^{\frac{k}{mc_p} t} S_a(t) \right) dt + C \quad (12)$$

where C is the integration constant determined by the initial condition.

Whereas:

$$T_w(t) = e^{-\frac{k}{mc_p} t} \left(\int \left(e^{\frac{k}{mc_p} t} S_p(t) + k e^{\frac{k}{mc_p} t} S_a(t) \right) dt + C \right) \quad (13)$$

Substitute the initial conditions $T_w(0) = T_0$ to find C:

$$T_0 = C,$$

therefore:

$$T_w(t) = e^{-\frac{k}{mc_p} t} \left(\int_0^t \left(e^{\frac{k}{mc_p} \tau} S_p(\tau) + k e^{\frac{k}{mc_p} \tau} S_a(\tau) \right) d\tau + T_0 \right) \quad (14)$$

Approximation of functions by splines

To construct splines, it is necessary to determine the coefficients a_i, b_i, c_i, d_i for each interval $[t_i, t_{i+1}]$ from the conditions of continuity, smoothness and coincidence of the function and its derivatives at the nodal points [9,11,12].

To interpolate the functions $T_a(t)$ and $P(t)$, we use the cubic spline algorithm:

1. Set the values of the function at the nodal points: $T_a(t)$ and $P(t)$.
2. We determine the values of the first and second derivatives at the nodal points.
3. We solve a system of linear equations to determine the spline coefficients.

Conclusion. In this article, we have considered the task of optimizing energy consumption in controlling the water temperature in the capacity of a smart home system. A mathematical model was built, taking into account heat transfer and temperature dynamics, and external parameters were approximated by splines. The integration of splines into the mathematical model made it possible to more accurately describe the dynamics of the system and minimize energy consumption while maintaining a set temperature.

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